Meeting Time: (2 x 1.5 hrs / Week)

Tuesdays: 1:30p – 3:00p (Williams hall, room 438)

Fridays: 3:00p - 4:30p (Towne, room 305)

[Nov 16th - Dec 11th] (7 classes – No class on Friday, Nov 23, due to Thanksgiving break).

Lecturer: Subhrajit Bhattacharya (subhrabh@math.upenn.edu). Office: DRL 3C7.

(c) Configuration spaces and topology

Background: Point-set topology (1 to 2 lectures):

- Motivation from robot configuration space (introduction to $S^1=SO(2)$, $S^1xS^1=T^2$, SE(2), $SO(3)\neq S^2xS^1$, SE(3), with examples)
- Definitions: Topology, continuous function, homeomorphism, embedding, immersion ... with examples (e.g. doughnut-coffeemug, etc)
- Construction of one topological spaces from other: Gluing (example: representation of torus, klein bottle, sphere)
- Definitions: Homotopy, isotopy ... example: homotopic trajectories, knots
- Definition: Manifolds, manifolds with boundary (example of topological spaces that are manifolds and that are not manifolds)

Basic algebraic topology (2 to 3 lectures):

- Motivation: why do we need this? - to be able to do computation; to obtain global information from local informations. Especially interesting when we can't 'visualize' the space.
- Introduction: Quick review of linear algebra (with emphasis on linear transformation and kernel)
- Simplicial complex, chain complex (illustrate with simple example), delta complex (use example of torus - illustrates computation of homology of torus later)
- Homology of a chain complex
- Quick introduction to cohomology
- Differential forms and De Rham cohomology

Application of algebraic topology in robotics (<1 lecture):

- Identification of "holes" in coverage by sensor networks (introduce Cech complex).
- Planning with topological constraints

* Few additional topics if time permits. (*Topology*: Cartesian product of spaces, Basic concept of fiber bundles, tangent bundle, Groups, Lie Groups; *Alg. Topology*: Long exact sequence; *Basic Riemannian geometry*).

Project ideas:

a) Identification of "holes" in coverage by sensor networks on R^3 (e.g. quadrotors) or S^2 (e.g. planetary exploration). Design control to attain full coverage (Similar to Jason's work - Integrates with Koushil's component).

   - ***Ref*:** J. Derenick, V. Kumar, A. Jadbabaie, "Towards simplicial coverage repair for mobile robot teams", in 2010 IEEE International Conference on Robotics and Automation (ICRA).
     URL: http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=05509808
   - Related exercise in Algebraic topology: Compute homology groups and their generators of some simple, yet nontrivial spaces in MATLAB (e.g. S^3, T^3, RP^2, SO(3), etc)

b) Search-based planning on SO(3), SE(3), etc. Planning in different homology classes or finding shortest trajectories in different homology classes? (Integrates with Steve's component)

   - ***Ref*:** S. Bhattacharya, M. Likhachev, V. Kumar (2012) "Topological Constraints in Search-based Robot Path Planning", Autonomous Robots, 33(3):273-290, October, Springer Netherlands.
     URL: http://www.springerlink.com/content/f851813u657g5923/
   - Related exercise in Algebraic topology: Procedures/functions to compute H-signatures (homology class invarients) of trajectories in C++ or MATLAB.

Text book:

- Topology by James Munkres (Prentice Hall, 2nd edn., 2000)
- Algebraic Topology by Allen Hatcher (available at http://www.math.cornell.edu/~hatcher/AT/ATpage.html)

Extra readings:

- Supplementary notes provided in class.
- Fiber Bundles by Dale Husemoller (Springer-Verlag, 1966)